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Digital Receiver Decision Techniques for Certain Fixed Length Binary Block Codes Transmitted Through the Gaussian Channel

NOEL C. [illegible] AND HUGO M. BECK

Radio Communication Systems Branch
Communications Sciences Division

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NAVAL RESEARCH LABORATORY
Washington, D.C.

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ABSTRACT

Various types of fixed length binary block encodings of binary data are presented. Binary signaling is used to transmit the binary encoded data through an additive white gaussian noise channel. Certain block decision techniques are investigated and compared. For certain block encoding schemes, the notion of an intelligent receiver is developed.

This study is considered basic in analyzing the use of redundant encoding in digital communications systems.

PROBLEM STATUS

This is an interim report; work on the problem is continuing.

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DIGITAL RECEIVER DECISION TECHNIQUES FOR CERTAIN FIXED LENGTH BINARY BLOCK CODES TRANSMITTED THROUGH THE GAUSSIAN CHANNEL

INTRODUCTION

A study was undertaken to provide some of the background basic to the understanding and intelligent pursuit of problems involving the use of redundant encoding schemes in digital communication systems.

Binary data emitted from a source is encoded into fixed length (N) blocks of bits according to various types of block encoding schemes. By using each of the N bits in the block to be transmitted to binary modulate a carrier in phase or frequency, N bit signals are produced and these are transmitted as the signal for that block. Depending on whether the binary signaling is coherent (C2PSK, C2FSK) or noncoherent (NC2FSK), i.e., depending on the bit coherency, and the availability of coherency over the block, the receiver processes the received bit signals according to various block decision techniques. Diagrams of receiver implementations for these block decision techniques are given. The probability of a block error (P_{eB}) is derived for the various block decision techniques. Using various values of N and E_b/N_o - the average signal energy per bit divided by the average noise power per unit bandwidth, the P_{eB} curves are plotted for each decision technique as a function of N and E_b/N_o. The block decision techniques are compared in terms of processing gain per bit for given values of N. For certain types of encodings the notion of an intelligent receiver is developed and some of its uses are discussed. Areas for further investigation are outlined.

ENCODING AND TRANSMISSION

Block Encoding Schemes

We assume that the source emits sequences of binary data elements where these may, for example, represent teletype characters or quantized analog signal samples.

Each data element emitted by the source is encoded into a block of N bits (N even, positive). Due to the binary nature of the data, for each data element the encoder must form a block B_0 which is to be transmitted if the data element is a zero and a block B_1 which is to be transmitted if the data element is a one. We let the sequences $\{b_{0i}\}$ $i = 1, \ldots, N$ and $\{b_{1i}\}$ $i = 1, \ldots, N$ represent the B_0 and B_1 blocks, respectively where $b_{ji} \in \{0, 1\}$ for all $i = 1, \ldots, N$ and $j = 0, 1$.

Next we characterize the important properties of the B_0 and B_1 blocks. In this connection we let $\rho(B_0, B_1)$ denote the un-normalized correlation between the B_0 and B_1 blocks, i.e., $\rho(B_0, B_1)$ equals the number (A) of bits i, such that $b_{0i} = b_{1i}$ minus the number (D) of bits i, such that $b_{0i} \neq b_{1i}$. Clearly $A + D = N$.

We say that the B_0 and B_1 blocks belong to a complementary block encoding scheme if $b_{0i} = b_{1i} \oplus 1$ (where $\oplus$ denotes addition modulo two) or equivalently if $\rho(B_0, B_1) = -N$. For this encoding scheme there are

$\sum_{K=1}^{N} \binom{N}{K} = 2^N - 1$ distinguishable ways to form B_0 and B_1. When N = 8, B_0 = 01000101 and B_1 = 10111010 is such a distinguishable complementary encoding.

If the B_0 and B_1 blocks belong to a complementary block encoding scheme and either $b_{0i} = 0$ and $b_{1i} = 1$ or $b_{0i} = 1$ and $b_{1i} = 0$ for all i = 1, ..., N, then we say that the B_0 and B_1 blocks belong to a simple complementary block encoding scheme. For this encoding scheme there is only one distinguishable way to form B_0 and B_1. When N = 8, then B_0 = 00000000 and B_1 = 11111111 is such a simple complementary encoding.

Let $N_0(B_j)$ and $N_1(B_j)$ denote the number of zeros and ones, respectively in the block B_j.

If the B_0 and B_1 blocks belong to a complementary block encoding scheme and we have $N_0(B_0) = N_1(B_0) = N_0(B_1) = N_1(B_1) = N/2$, then we say that the B_0 and B_1 blocks belong to an equi-distributed complementary block encoding scheme. For this scheme there are $\binom{N}{N/2} = N!/[(N/2)!]^2$ distinguishable ways to form B_0 and B_1. When N = 8, B_0 = 01101010 and B_1 = 10010101 is such a distinguishable equi-distributed complementary encoding.

Let N_{jk} denote the number of bits i for which $b_{0i} = j$ and $b_{1i} = k$ where j = 0, 1 and k = 0, 1 and i = 1, ..., N.

Suppose that N is such that N/4 is an integer. We say that the B_0 and B_1 blocks belong to an equi-distributed random orthogonal block encoding scheme if:

(i) $N_0(B_0) = N_1(B_0) = N_0(B_1) = N_1(B_1) = N/2$ and
(ii) $\rho(B_0, B_1) = 0$.

Clearly for this scheme $A = D = N/2$ and $N_{00} = N_{01} = N_{10} = N_{11} = N/4$. Also, for this scheme there are $\binom{N}{N/2} \cdot \binom{N/2}{N/4} \cdot \binom{N/2}{N/4} = N!/[(N/4)!]^4$ distinguishable ways to form B_0 and B_1. When N = 8, B_0 = 01110100 and B_1 = 01100011 is such an equi-distributed random orthogonal encoding.

If the B_0 and B_1 blocks belong to an equi-distributed random orthogonal block encoding scheme and we form blocks $\hat{B}_0$ and $\hat{B}_1$ from B_0 and B_1, respectively by using only the $D = N_{01} + N_{10} = N/2$ bits where B_0 and B_1 disagree, then $\hat{B}_0$ and $\hat{B}_1$ will belong to an equi-distributed complementary block encoding scheme of length $\hat{N} = N/2$.

Given a particular block encoding scheme we allow that the B_0 and B_1 blocks chosen from this scheme can be the same for all binary data elements, vary for each data bit according to a deterministic rule, or vary for each data bit according to some discrete probability distribution.

Modulation and the Channel

Now suppose the source emits a data bit which in turn determines through a particular block encoding scheme the block $B_j = \{b_{j,i}\} = 1, \ldots, N$ to be transmitted. The bits $b_{j,i}$ of B_j binary modulate a signal $S(t)$ in frequency or phase producing bit signals $X_{j,i}(t)$ where for all i, $X_{j,i}(t)$ belongs to a binary PSK or FSK signal set and each $X_{j,i}(t)$ has average energy E_b.

Hence, the block B_j determines a block signal $X(t) = \{X_{j,i}(t)\}$ $i = 1, \ldots, N$ which is transmitted bit signal by bit signal through a channel, which we assume adds white gaussian noise of one-sided spectral density N_0 to each bit signal. If T_b is the bit time and W is the effective receiver bandwidth, then we will assume throughout it follows that $T_b W = 1$.

RECEPTION AND DECISION TECHNIQUES

Now we consider certain block decision techniques for various block encoding schemes and binary signal sets. We assume that the B_0 and B_1 blocks are available at the receiver.

The Bit by Bit Block Decision Technique

In this decision technique we employ for each bit signal $X_{j,i}(t)$ a binary signal detection meth ' the form of which depends on the modulation used and the available bit phase coherency at the receiver. After the detection procedure we make a bit decision on which signal of the binary signal set was transmitted. This decision corresponds to a binary decision d_i on $b_{j,i}$. After completing this for all N bit signals, we have formed a block $D = \{d_i\}$ $i = 1, \ldots, N$ which is the receiver's estimate of the transmitted block $B_j = \{b_{j,i}\}$ $i = 1, \ldots, N$. We make a block decision by computing $\rho(D, B_0)$ and $\rho(D, B_1)$ and then deciding that B_0 was transmitted if $\rho(D, B_0) > \rho(D, B_1)$ and that B_1 was transmitted if $\rho(D, B_1) > \rho(D, B_0)$. If $\rho(D, B_0) = \rho(D, B_1)$, then we flip an unbiased coin to reach a decision.

Diagram 1 gives an implementation of the bit by bit block decision technique receiver when NC2FSK signaling is employed with logic corresponding to frequency.

Now ... probability of a block error, P_{eB}, for a given N and E_b/N_0 using the bit by bit decision technique is:

$$P_{eB} = \begin{cases} \sum_{K=\frac{N+1}{2}}^{N} \binom{N}{K} P_{eb}^{K} (1 - P_{eb})^{N-K}, & N \text{ odd} \\ \sum_{K=\frac{N}{2}+1}^{N} \binom{N}{K} P_{eb}^{K} (1 - P_{eb})^{N-K} + \frac{1}{2} \binom{N}{\frac{N}{2}} P_{eb}^{\frac{N}{2}} (1 - P_{eb})^{\frac{N}{2}}, & N \text{ even} \end{cases}$$

P_{eb} is the probability of a bit error, i.e., $P(d_i \neq b_{j,i}) = P_{eb}$ and thus clearly, P_{eb} depends on the type of signaling employed. It is well known that for white gaussian noise of one-sided spectral density N_o, P_{eb} equals $\frac{1}{2}[1-\text{erf}(\sqrt{E_b/N_o})]$ for C2PSK signaling, $\frac{1}{2}[1-\text{erf}(\sqrt{E_b/2N_o})]$ for C2FSK signaling and $\frac{1}{2}\exp[-E_b/2N_o]$ for NC2FSK signaling where

$$\text{erf}(X) = \frac{2}{\sqrt{\pi}} \int_0^X \exp[-y^2]dy.$$

For C2FSK and NC2FSK signaling we assume that the modulation index is $m = \frac{1}{2}$ or $m = 1$.

The P_{eB} curves for C2PSK, C2FSK, and NC2FSK signaling are given in Figures 1, 2, and 3, respectively. These curves we computed on NRL's CDC 3800 computer using the fact that for a given signaling type, E_b/N_o and N odd, $P_{eB}(N) = P_{eB}(N + 1)$.

Next we consider for which block encoding schemes these P_{eB} curves hold. Suppose that the B_o and B_1 blocks are such that $-N < \rho(B_o, B_1) < N$. Clearly a bit error $(d_i \neq b_{j,i})$ will influence P_{eB} only when $b_{o,i} \neq b_{1,i}$, since if $(b_{o,i} = b_{1,i}) \neq d_i$, then both $\rho(D, B_o)$ and $\rho(D, B_1)$ have a minus one contribution for that bit. Hence, the P_{eB} curves using the formula given above hold only for complementary, simple complementary, or equi-distributed complementary block encoding schemes. If the B_o and B_1 blocks (of length N) belong to an equi-distributed random orthogonal block encoding scheme, then we see from the considerations above that the P_{eB} curves for this scheme are given by the curves of Figures 1, 2, and 3 where the redundancy is $\hat{N} = N/2$.

The NC2FSK-Sum Block Decision Technique

For this decision technique we assume that binary FSK modulation is employed with modulation index $m = \frac{1}{2}$ or $m = 1$ such that frequency corresponds to logic and due to a lack of bit phase knowledge, we must for each bit signal $X_{j,i}(t)$ detect the envelopes $E_{o,i}$ and $E_{1,i}$ at the binary signaling frequencies. Then using B_o and B_1 blocks we form for B_o and B_1 $\mathcal{E}(B_o)$ and $\mathcal{E}(B_1)$, respectively where $\mathcal{E}(B_k)$ is a sum of bit envelopes $Z_{k,i}$ such that the envelopes $Z_{k,i}$ are those which would contain the signal had the B_k block been transmitted. This summing of appropriate bit envelopes is a method of post-detection integration. We make a block decision by

deciding B_0 was transmitted if $\mathcal{E}(B_0) > \mathcal{E}(B_1)$ and that B_1 was transmitted if $\mathcal{E}(B_1) > \mathcal{E}(B_0)$. If $\mathcal{E}(B_0) = \mathcal{E}(B_1)$, then we flip an unbiased coin to reach a decision. From references 1 and 2, we see that this technique as described above is optimum for high E_b/N_0 ratios and the optimum form of this technique for low E_b/N_0 ratios is obtained by replacing the bit envelopes $E_{0,i}$ and $E_{1,i}$ by $E_{0,i}^2$ and $E_{1,i}^2$.

Diagram 2 gives an implementation of the NC2FSK-Sum Block Decision Technique receiver for low E_b/N_0.

Given N and E_b/N_0 (low E_b/N_0 ratio) and assuming a complementary, simple complementary or equi-distributed complementary block encoding scheme, we have, from Appendix 1 assuming the B_0 block was transmitted, that $\mathcal{E}(B_0)$ is determined to within a constant C by a non-central Chi-Squared distribution with 2N degrees of freedom and non-central parameter $2N\,E_b/N_0$ and $\mathcal{E}(B_1)$ is determined to within the same constant C by a Chi-Squared distribution with 2N degrees of freedom. Using these distributions and the approximations found in reference 3, the P_{eB} curves, given in Figure 4, were computed on NRL's CDC 3800 computer using function subprograms for erf(X) and the inverse cumulative distribution function of the Chi-Squared distribution with $L \geq 1$ degrees of freedom.

Now suppose the B_0 and B_1 blocks (of length N) belong to an equi-distributed random orthogonal block encoding scheme. Since in this case $Z_{0,i} = Z_{1,i}$ for the bits i where the blocks agree, the bits where the blocks agree do not affect the block decision and hence the P_{eB} curves for this scheme are given by the curves of Figure 4 where the redundancy is $\hat{N} = N/2$.

The NCB2FSK-Add Block Decision Technique

For this decision technique we assume that binary FSK modulation is employed with modulation index m = 1, frequency corresponds to logic, and there is phase coherency over the block such that for either frequency the initial phase of all N possible bit signals at that frequency is the same but this value is unknown. Let f_0 and f_1 denote the binary signaling frequencies.

The decision technique forms for each bit, the quadrature components at the binary signaling frequencies ; i.e., for the i^{th} bit the receiver forms $X(i,s,f_0)$, $X(i, c, f_0)$, $X(i, s, f_1)$ and $X(i, c, f_1)$. Due to the phase coherency over the block with respect to f_0 and f_1 and since frequency corresponds to logic; i.e., $b_{j,i} = 0$ corresponds to f_0 and $b_{j,i} = 1$ corresponds to f_1 $(j = 0, 1)$, we can employ predetection integration and form for block B_j $(j = 0, 1)$:

$Y(B_j, s, f_0) = \Sigma\, X(i, s, f_0)$, $Y(B_j, c, f_0) = \Sigma\, X(i, c, f_0)$, where the summation is over i s.t. $b_{j,i} = 0$ with $i = 1, \ldots, N$; and

$Y(B_j, s, f_1) = \Sigma\, X(i, s, f_1)$, $Y(B_j, c, f_1) = \Sigma\, X(i, c, f_1)$, where the summation is over i s.t. $b_{j,i} = 1$ with $i = 1, \ldots, N$.

Now since the initial phase of any bit signal at either frequency is unknown, we form for block B_j $(j = 0, 1)$ the block type envelope at each frequency; i.e., we form $E(B_j, f_o) = \sqrt{Y^2(B_j, s, f_o) + Y^2(B_j, c, f_o)}$ and $E(B_j, f_1) = \sqrt{Y^2(B_j, s, f_1) + Y^2(B_j, c, f_1)}$.

Then we form for B_j $(j = 0, 1)$, $\mathcal{E}(B_j) = E(B_j, f_o) + E(B_j, f_1)$ and we make a block decision by deciding that B_o was transmitted if $\mathcal{E}(B_o) > \mathcal{E}(B_1)$ and that B_1 was transmitted if $\mathcal{E}(B_1) > \mathcal{E}(B_o)$. If $\mathcal{E}(B_o) = \mathcal{E}(B_1)$ then we flip an unbiased coin to reach a decision. From References 1 and 2 we see that this technique as described above is optimum for high E_b/N_o ratios. For low E_b/N_o ratios, the optimum form of this technique is obtained by replacing the block type envelopes $E(B_j, f_o)$ and $E(B_j, f_1)$ by $E^2(B_j, f_o)$ and $E^2(B_j, f_1)$ $(j = 0, 1)$.

Diagram 3 gives an implementation of the NCB2FSK-Add Block Decision Technique receiver for low E_b/N_o ratios.

Next we consider the P_{eB} curves resulting for the various block encoding schemes assuming low E_b/N_o ratios. Suppose that the B_o and B_1 blocks (of length N) belong to a simple complementary block encoding scheme. Let us suppose that for all $i = 1, \ldots, N$, $b_{oi} = 0$ and $b_{1i} = 1$. In this case $Y(B_o, s, f_1) = Y(B_o, c, f_1) = Y(B_1, s, f_o) = Y(B_1, c, f_o) = 0$ and hence $\mathcal{E}(B_o) = E^2(B_o, f_o)$ and $\mathcal{E}(B_1) = E^2(B_1, f_1)$. We see that for simple complementary block encoding schemes the decision for high or low E_b/N_o ratios results in the same P_{eB} curves. From Reference 1 we see that for a given N and E_b/N_o, the probability of a block error is given by:

$$P_{eB} = \tfrac{1}{2} \exp[-NE_b/2N_o]$$

These curves are given in Figure 5 and were computed on a Wang 600 Series calculator.

Suppose now that the B_o and B_1 blocks (of length N) belong to an equi-distributed complementary block encoding scheme. From Appendix 2, assuming the B_o block was transmitted, we have that $\mathcal{E}(B_o)$ is determined to within a constant C by a non-central Chi-Squared distribution with 4 degrees of freedom and non-central parameter $2N E_b/N_o$ and $\mathcal{E}(B_1)$ is determined to within the same constant C by a Chi-Squared distribution within 4 degrees of freedom. Using these distributions and the approximations found in Reference 4, the P_{eB} curves, given in Figure 6, were computed on NRL's CDC 6600 computer using function subprograms for erf(X) and the inverse cumulative distribution function of the Chi-Squared distribution with K≥1 degrees of freedom.

Suppose next that the B_o and B_1 blocks (of length N) belong to a complementary block encoding scheme. Comparing the P_{eB} curves assuming simple complementary encoding to the P_{eB} curves assuming equi-distributed complementary encoding and noting the nature of complementary encoding in terms of these limiting cases we see that for a given N,

the P_{eB} curve for complementary encoding lies between the P_{eB} curve for simple complementary encoding at N and the P_{eB} curve for equi-distributed complementary encoding at N. Exactly where the P_{eB} curve for complementary encoding lies depends on the distribution of ones and zeros in either the B_0 or B_1 block. The more nearly uniform this distribution is the closer the P_{eB} curve for complementary encoding approaches to the P_{eB} curve for equi-distributed complementary encoding and the more nearly this distribution resembles the degenerate distribution the closer the P_{eB} curve for complementary encoding approaches to the P_{eB} curve for simple complementary encoding.

Suppose finally that the B_0 and B_1 blocks (of length N) belong to an equi-distributed, random orthogonal block encoding scheme. In this case, we see from the properties of this type of encoding scheme that $Y(B_0, s, f_0)$ and $Y(B_0, c, f_0)$ will have N/4 terms in common with $Y(B_1, s, f_0)$ and $Y(B_1, c, f_0)$, respectively and $Y(B_0, s, f_1)$ and $Y(B_0, c, f_1)$ will have N/4 terms in common with $Y(B_1, s, f_1)$ and $Y(B_1, c, f_1)$, respectively. Hence, $E^2(B_0, f_0)$ and $E^2(B_1, f_0)$ will each consist of a block type squared envelope term where the quadrature components appearing therein are taken with respect to the complementary bit portions of the blocks plus a term which is the same for both $E^2(B_0, f_0)$ and $E^2(B_1, f_0)$ plus a term consisting of a product of two sums of quadrature components where one sum is the same for both $E^2(B_0, f_0)$ and $E^2(B_1, f_0)$. The same relation exists between $E^2(B_0, f_1)$ and $E^2(B_1, f_1)$. If we regard the block decision as consisting only of a comparison of the sums of the block type squared envelopes, then, since these are formed only with respect to the complementary bit portions of the blocks, the resulting P_{eB} curves would be the same as those for equi-distributed complementary block encoding where the redundancy is $\bar{N} = N/2$. But because of the extra terms, which are just normal random variables appearing in $E^2(B_0, f_0)$ and $E^2(B_1, f_0)$ and $E^2(B_0, f_1)$ and $E^2(B_1, f_1)$, we see that the P_{eB} curves for equi-distributed random orthogonal block encoding schemes (of length N) are slightly to the left of the P_{eB} curves (of length N/2) for equi-distributed complementary block encoding schemes.

The NCBFSK-Add, Subtract Block Decision Technique

For this decision technique we assume that binary FSK modulation is employed with modulation index m = 1, frequency corresponds to logic, and there is phase coherency over the block such that for either frequency the initial phase of all N possible bit signals at that frequency is the same but this value is unknown. Let f_0 and f_1 denote the binary signaling frequencies.

The decision technique forms for each bit, the quadrature components at the binary signaling frequencies; i.e., for the i^{th} bit the receiver forms $X(i, s, f_0)$, $X(i, c, f_0)$, $X(i, s, f_1)$, and $X(i, c, f_1)$.

Due to the phase coherency over the block and since frequency corresponds to logic, we can employ predetection integration and form for B_j $(j = 0, 1)$:

$$Z(B_j, s, f_o) = \sum_{i=1}^{N} (-1)^{b_{j,i}} X(i, s, f_o)$$

$$Z(B_j, c, f_o) = \sum_{i=1}^{N} (-1)^{b_{j,i}} X(i, c, f_o)$$

$$Z(B_j, s, f_1) = \sum_{i=1}^{N} (-1)^{[b_{j,i} + 1]} X(i, s, f_1)$$

$$Z(B_j, c, f_1) = \sum_{i=1}^{N} (-1)^{[b_{j,i} + 1]} X(i, c, f_1)$$

Now since the initial phase of any bit signal at either frequency is unknown, we form for block B_j $(j = 0, 1)$ the block type envelopes at each frequency; i.e., we form $E(B_j, f_o) = \sqrt{Z^2(B_j, s, f_o) + Z^2(B_j, c, f_o)}$ and $E(B_j, f_1) = \sqrt{Z^2(B_j, s, f_1) + Z^2(B_j, c, f_1)}$. Then we form for B_j $(j = 0, 1)$, $\mathcal{E}(B_j) = E(B_j, f_o) + E(B_j, f_1)$ and we make a block decision by deciding that B_o was transmitted if $\mathcal{E}(B_o) > \mathcal{E}(B_1)$ and that B_1 was transmitted if $\mathcal{E}(B_1) > \mathcal{E}(B_o)$. If $\mathcal{E}(B_o) = \mathcal{E}(B_1)$, then we flip an unbiased coin to reach a decision. From References 1 and 2, we see that this technique as described above is optimum for high E_b/N_o ratios. For low E_b/N_o ratios, the optimum form of this technique is obtained by replacing the block type envelopes $E(B_j, f_o)$ and $E(B_j, f_1)$ by $E^2(B_j, f_o)$ and $E^2(B_j, f_1)$ $(j = 0, 1)$.

Diagram 3 gives an implementation of the NCB2FSK-Add, Substract Block Decision Technique receiver for low E_b/N_o with $Y(B_j, s, f_k)$ and $Y(B_j, c, f_k)$ replaced by $Z(B_j, s, f_k)$ and $Z(B_j, c, f_k)$ for $j = 0, 1$ and $k = 0, 1$. Given N and E_b/N_o (low E_b/N_o ratio) and assuming an equi-distributed random orthogonal block encoding scheme, we have from Appendix 3, assuming the B_o block was transmitted, that $\mathcal{E}(B_o)$ is determined to within a constant C by a non-central Chi-Squared Distribution with 4 degrees of freedom and non-central parameter $N E_b/N_o$ and that $\mathcal{E}(B_1)$ is determined to within the same constant C by a Chi-Squared Distribution with 4 degrees of freedom. Comparing these distributions for $\mathcal{E}(B_o)$ and $\mathcal{E}(B_1)$ with those for the equi-distributed complementary case of the NCB2FSK-Add Block Decision Technique given above we find that for this decision technique the P_{eB} curves for B_o and B_1 blocks (of length N) belonging to an equi-distributed random orthogonal block encoding scheme are given by the P_{eB} curves for the equi-distributed complementary block encoding case of the NCB2FSK-Add Block Decision Technique at a redundancy $\hat{N} = N/2$.

The CB2FSK Block Decision Technique

For this decision technique we assume that binary FSK modulation is employed with modulation index m = 1, frequency corresponds to logic, and there is phase coherency over the block such that for either frequency the initial phase of all possible bit signals at that frequency is the same and this value is known.

If we assume that the B_0 and B_1 blocks (of length N) belong to a simple complementary block encoding scheme, then from Reference 6 we see that we can optimally matched filter process the received block signal with the resulting P_{eB} for a given N and E_b/N_o being given by: $P_{eB} = \frac{1}{2}[1-\text{erf}(\sqrt{N E_b/2N_o})]$. These curves are given in Figure 7 and were computed on a 300 Series Wang calculator using Reference 5.

The CB2PSK Block Decision Technique

For this decision technique we assume that binary PSK modulation is employed and there is phase coherency over the block such that for either bit signal the initial phase is the same and this value is known for N repetitions of the bit signal.

If we assume that the B_0 and B_1 blocks of length N belong to a simple complementary block encoding scheme, then we can optimally process the received block signal with the resulting P_{eB} for a given N and E_b/N_o being given by: $P_{eB} = \frac{1}{2}[1-\text{erf}(\sqrt{N E_b/N_o})]$. These curves are given in Figure 8 and were computed on a 300 Series Wang calculator using Reference 5.

A Comparison of Decision Techniques

Suppose binary FSK modulation is employed with modulation index $m = \frac{1}{2}$ or 1, frequency corresponds to logic and the initial phase of each of the N possible bit signals at either frequency is different and this value is unknown. Clearly one can employ only the NC2FSK signaling case of the Bit by Bit Decision Technique or the NC2FSK-Sum Block Decision Technique in making a block decision. If we compare these alternatives, then from Figures 3 and 4 we see that for any given N, the P_{eB} curve for the NC2FSK-Sum Block Decision Technique show a processing gain advantage of approximately 2 dB per bit with respect to the P_{eB} curves for the NC2FSK signaling case of the Bit by Bit Decision Technique. We note that this comparison holds for all types of block encoding schemes.

Now suppose that m = 1 and the initial phase of each of the N possible bit signals at either frequency is the same but this value is unknown. This allows us to employ the NCB2FSK-Add Block Decision Technique in making a block decision. If we recall the relation between the P_{eB} curves for the different types of block encoding schemes for this

decision technique and then examine Figures 5 and 6, then we see that the P_{eB} curves of Figures 5 and 6 bound the variation for the different types of block encoding schemes for the NCB2FSK-Add Block Decision Technique except we must remember that for the equi-distributed random orthogonal block encoding schemes we have an effective redundancy of $\hat{N} = N/2$, thus these P_{eB} curves at a redundancy of N are bounded by the P_{eB} curves of Figures 5 and 6 for a redundancy of $\hat{N} = N/2$.

If we compare, from a processing gain advantage per bit standpoint, the NCB2FSK-Add Block Decision Technique with the NC2FSK-Sum Block Decision Technique using Figures 4, 5, and 6, then we see that this advantage will be a function of N, the block encoding scheme employed and the value of P_{eB} chosen as a reference. For any block encoding scheme and any value of P_{eB}, we see that the NCB2FSK-Add Block Decision Technique has a processing gain advantage over the NC2FSK-Sum Block Decision Technique for any $N \geq 4$ and this advantage clearly increases as N increases. For example, if we employ equi-distributed complementary encoding and set $P_{eB} = 10^{-3}$, then for $N = 4$ the advantage is about 1 dB per bit whereas for $N = 2048$ the advantage is about 12 dB per bit. Using these comparisons of the NCB2FSK-Add technique with the NC2FSK-Sum technique we can, using the results previously obtained, easily extend these results to comparisons of the NC2FSK Bit by Bit technique with the NCB2FSK-Add technique.

Next let us suppose that we can employ coherent signaling per bit and there is phase coherency over the block with respect to each of the bit signals in the binary signaling set. If the block encoding scheme can be any of the various types mentioned, we must employ the Bit by Bit Decision Technique. Due to the fact that we can employ coherent signaling per bit, we can utilize either the C2PSK case or the C2FSK case of the Bit by Bit Decision Technique. If we compare these alternatives, then from Figures 1 and 2 we see that for any N and any type of block encoding scheme, the P_{eB} curves for the C2PSK case show a processing gain advantage of approximately 3 dB per bit with respect to the P_{eB} curves for the C2FSK case.

Now if we restrict the block encoding schemes to the simple complementary block encoding scheme, then we can employ the CB2FSK and CB2PSK Block Decision Techniques. If we compare these alternatives, then from Figures 7 and 8 we see that for any N the P_{eB} curves for the CB2PSK technique show a processing gain advantage of 3 dB per bit with respect to the P_{eB} curves for the CB2FSK technique. If we compare the CB2PSK technique and the C2PSK Bit by Bit technique or the CB2FSK technique and the C2FSK Bit by Bit technique, then we see from Figures 1 and 8 and 2 and 7 that the Bit by Bit technique shows approximately a 2 dB loss in processing gain per bit versus the corresponding technique for any N and when simple complementary block encoding is employed.

THE INTELLIGENT RECEIVER

Let N_{jk} denote the number of bits i for which $b_{0i} = j$ and $b_{1i} = k$ where $j = 0, 1$ and $k = 0, 1$ and $i = 1, \ldots, N$. Suppose we employ a block encoding scheme which has the property that, for any pair of B_0 and B_1 blocks belonging to this scheme, $N_{11} \neq 0$ and $N_{00} \neq 0$. Clearly an equi-distributed random orthogonal block encoding scheme has this property since $N_{00} = N_{11} = N/4$, but a simple complementary, complementary, or equi-distributed complementary block encoding scheme does not have this property. Now suppose that, in addition to implementing a particular block decision technique, for each of the $N_{00} + N_{11}$ bits where the blocks agree, the receiver makes a hard decision on which of the bit signals was transmitted. Thus, the receiver makes a hard bit decision d_i on the transmitted bit b_{ji} for these $N_{00} + N_{11}$ bits i. Due to the fact that the B_0 and B_1 blocks are available at the receiver, the receiver knows precisely the value of b_{ji} for these $N_{00} + N_{11}$ bits i. Hence, the receiver can compute $r_0 = \Sigma\, d_i$, where the summation is over i s.t. $b_{0i} = b_{1i} = 0$ with $i = 1, \ldots, N$; and $r_1 = \Sigma\, (d_i \oplus 1)$ where the summation is over i s.t. $b_{0i} = b_{1i} = 1$ with $i = 1, \ldots, N$, which represent the number of obvious bit errors with respect to the N_{00} and N_{11} bits, respectively. Assuming a block encoding scheme is employed where $N_{00} \neq 0$ and $N_{11} \neq 0$, a receiver which, in addition to implementing a particular block decision technique, operates in a manner described above to compute the numbers r_0, r_1 and N_{00}, N_{11} will be called an intelligent receiver. An implementation of the intelligent portion of the receiver for NC2FSK signaling is given in Diagram 4.

Now let us examine the utilizations of the intelligent receiver. Basically we can distinguish two types of utilizations of the intelligent receiver. These involve either direct or indirect use of the measured values r_0, r_1 and N_{00}, N_{11}.

An example of a direct use of the measured values would be using these values to measure the actual probability of a bit error. $\hat{P}_{eb}$ where $\hat{P}_{eb}$ is simply the relative frequency of all obvious bit errors, i.e., for a particular data bit transmission $\hat{P}_{eb} = \frac{r_0 + r_1}{N_{00} + N_{11}}$. From Reference 7, pages 191-209, we see that for $N_{00} + N_{11}$ sufficiently large it follows from the Laws of Large Numbers that $\hat{P}_{eb}$ converges to P_{eb} assuming the conditions used to determine P_{eb} are valid. If $N_{00} + N_{11}$ is not sufficiently large so that $\hat{P}_{eb}$ converges to P_{eb} we can form $\hat{P}_{eb}$ with respect to a number of data bit transmissions; i.e., we can add the r_0 and r_1 values for these transmissions together and add the N_{00} and N_{11} for these transmissions together and then determine the relative frequency $\hat{P}_{eb}$ with respect to these sums.

An example of an indirect use of the measured values would be using these values in testing a hypothesis concerning the nature of the noise in

the communication system. Suppose we let H_o be the hypothesis that the noise is white gaussian noise. In the case when H_o is true and given a binary signaling technique (C2PSK, C2FSK, or NC2FSK) we can determine P_{eb}. In order to test the hypothesis H_o we choose an error probabiliy α such that we want our test to decide correctly that H_o is true with probability $1 - \alpha$. We use the probability α to determine a threshold R such that if $r_{oo} + r_{11} \leq R$, then by choosing H_o to be true we can be correct with probability $1 - \alpha$. Thus, when we reject the hypothesis H_o for $r_{oo} + r_{11} > R$, we are incorrect only with probability α. Clearly R is a function of P_{eb}, α, and $N_{oo} + N_{11}$ and R is determined by the equation:

$$\sum_{K=0}^{R} \binom{N_{oo} + N_{11}}{K} P_{eb}^{K} (1 - P_{eb})^{N_{oo} + N_{11} - K} = 1 - \alpha$$

AREAS FOR FURTHER INVESTIGATION

In the area of redundant encoding techniques, concatenated or nested codes will be investigated since they show promise of high noise resistance. In the area of modulation and signaling techniques, multi-level or M-ary techniques such as MPSK, MFSK and combined modulation techniques will be investigated. A previous report (see Reference 9) will provide the background basic to this investigation. In the area of detection techniques, non-parametric detectors will be investigated due to their promise of a lower probability of error when the noise is not additive white gaussian noise.

It is also planned that physical implementation of these techniques will be pursued in the form of a real time digital receiver with ample consideration being given to software as well as hardware oriented digital receiver design.

We would like to thank J. W. Linnehan (NRL Code 5414) for his helpful attitude and constructive criticism in the preparation of this report, and Dorothy M. Creamer for her patient and expert typing help.

APPENDIX 1

For the low E_b/N_o ratio case of the NC2FSK-Sum Block Decision Technique we investigate the distribution of $\mathcal{E}(B_j)$ assuming that the B_i block was transmitted and that the B_j block was not transmitted. In what follows, we will use the symbols given in Diagram 2 and let s be the signal power, σ^2 be the average noise power and φ_i be the phase distortion of the signal for the i^{th} bit.

First let us suppose that the B_i block was not transmitted. It is well known (see Reference 1, page 168) that, for all i = 1, ..., N, $\sqrt{Z_{ji}}$ is a Rayleigh distributed random variable with density function $P(\sqrt{Z_{ji}};X)$ given by:

$$P(\sqrt{Z_{ji}};X) = \begin{cases} \frac{X}{\sigma^2} \exp[-X^2/2\sigma^2], & X > 0 \\ 0, & X \leq 0 \end{cases}$$

From Reference 8, page 79 it follows that the density function of Z_{ji} is:

$$P(Z_{ji};Y) = \begin{cases} \frac{1}{2\sigma^2} \exp[-Y/2\sigma^2], & Y > 0 \\ 0, & Y \leq 0 \end{cases}$$

Thus from Reference 8, page 173 we see that for all i = 1, ..., N, Z_{ji} is an exponential random variable with parameter $1/2\sigma^2$. Now $\mathcal{E}(B_j) = \sum_{i=1}^{N} Z_{ji}$ is the sum of N independently and identically distributed exponential random variables with parameter $1/2\sigma^2$. From Reference 8, pages 194-5 we have that the moment generating function of each Z_{ji} is given by:

$$M(Z_{ji};t) = \frac{\frac{1}{2\sigma^2}}{\frac{1}{2\sigma^2} - t} \quad \text{for } t < \frac{1}{2\sigma^2}$$

From Reference 8, page 203 we see that the moment generating function of $\mathcal{E}(B_j)$ is given by:

$$M(\mathcal{E}(B_j);\, t) = \left(\frac{\frac{1}{2\sigma^2}}{\frac{1}{2\sigma^2} - t}\right)^N \quad \text{for } t < \frac{1}{2\sigma^2}$$

Now $M(\mathcal{E}(B_j); t)$ is the moment generating function of the random variable $\mathcal{E}(B_j)$ where $\mathcal{E}(B_j)/\sigma^2$ has a Chi-Squared Distribution with 2N degrees of freedom. Thus $\mathcal{E}(B_j)$ is determined to within the constant $C = 1/\sigma^2$ by a Chi-Squared Distribution with 2N degrees of freedom.

Next let us suppose that the B_j block was transmitted. We consider first the distribution of $Z_{j,i}$. Clearly $Z_{j,i} = X^2(j, c, i) + X^2(j, s, i)$ where $X(j, c, i)$ and $X(j, s, i)$ are the quadrature components associated with B_j for the i^{th} bit. From Reference 1, page 168 we see that, since B_j was transmitted, $X(j, c, i)$ has a normal distribution with mean = $\sqrt{s} \cos \varphi_i$ and variance σ^2 and $X(j, s, i)$ has a normal distribution with mean = $\sqrt{s} \sin \varphi_i$ and variance = σ^2. From Reference 8, page 169 and Reference 4, pages 48-52 we see that $X^2(j, c, i)/\sigma^2$ and $X^2(j, s, i)/\sigma^2$ have a non-central Chi-Squared Distribution with one degree of freedom and non-central parameters $s \cos^2 \varphi_i/\sigma^2$ and $s \sin^2 \varphi_i/\sigma^2$, respectively; hence from the additive property $Z_{j,i}/\sigma^2 = X^2(j, c, i)/\sigma^2 + X^2(j, s, i)/\sigma^2$, has a non-central Chi-Squared Distribution with two degrees of freedom and non-central parameter $s/\sigma^2 = 2E_b/N_o$. Using the additive property and since the $Z_{j,i}$ are independently and identically distributed, we see that $\mathcal{E}(B_j)/\sigma^2 = \sum_{i=1}^{N} Z_{j,i}/\sigma^2$ has a non-central Chi-Squared distribution with 2N degrees of freedom and non-central parameter $2N\, E_b/N_o$. Thus $\mathcal{E}(B_j)$ is determined to within the constant $C = 1/\sigma^2$ by a non-central Chi-Squared distribution with 2N degrees of freedom and non-central parameter $2N\, E_b/N_o$.

APPENDIX 2

For the low E_b/N_o ratio case of the NCB2FSK-Add Block Decision Technique, with equi-distributed complementary block encoding, we investigate the distribution of $\mathcal{E}(B_j)$ assuming that the B_j block was transmitted and that the B_j block was not transmitted. In what follows we will use the symbols given in Diagram 3 and let s be the signal power, σ^2 be the average noise power and φ_o and φ_1 be the initial, unknown signal phases at f_o and f_1, respectively.

First let us suppose that the B_j block was not transmitted. From Reference 1, page 168 we see that the quadrature components appearing in $Y(B_j, s, f_o)$, $Y(B_j, c, f_o)$, $Y(B_j, s, f_1)$ and $Y(B_j, c, f_1)$ have normal distributions with mean = 0 and variance = σ^2. From Reference 8, page 200 it follows that $Y(B_j, s, f_o)$, $Y(B_j, c, f_o)$, $Y(B_j, s, f_1)$, and $Y(B_j, c, f_1)$ have normal distributions with mean = 0 and variance $N\sigma^2/2$. Thus from Reference 8, pages 169, 180 it follows that $Y^2(B_j, s, f_o)/(N\sigma^2/2)$, $Y^2(B_j, c, f_o)/(N\sigma^2/2)$, $Y^2(B_j, s, f_1)/(N\sigma^2/2)$ and $Y^2(B_j, c, f_1)/(N\sigma^2/2)$ have a Chi-Squared Distribution with one degree of freedom. Hence $E^2(B_j, f_o)/(N\sigma^2/2) = Y^2(B_j, s, f_o)/(N\sigma^2/2) + Y^2(B_j, c, f_o)/(N\sigma^2/2)$ and $E^2(B_j, f_1)/(N\sigma^2/2) = Y^2(B_j, s, f_1)/(N\sigma^2/2) + Y^2(B_j, c, f_1)/(N\sigma^2/2)$ have, from Reference 8, page 201, a Chi-Squared Distribution with two degrees of freedom. Using this additivity property again we see that $\mathcal{E}(B_j)/(N\sigma^2/2) = E^2(B_j, f_o)/(N\sigma^2/2) + E^2(B_j, f_1)/(N\sigma^2/2)$ has a Chi-Squared Distribution with four degrees of freedom. Thus $\mathcal{E}(B_j)$ is determined to within the constant $C = 2/N\sigma^2$ by a Chi-Squared Distribution with four degrees of freedom.

Next let us suppose that the B_j block was transmitted. From Reference 1, page 168, we see that the quadrature components appearing in $Y(B_j, s, f_o)$, $Y(B_j, c, f_o)$, $Y(B_j, s, f_1)$ and $Y(B_j, c, f_1)$ have a normal distribution with means = $\sqrt{s}\sin\varphi_o$, $\sqrt{s}\cos\varphi_o$, $\sqrt{s}\sin\varphi_1$ and $\sqrt{s}\cos\varphi_1$, respectively and variance = σ^2. From Reference 8, page 200 it follows that $Y(B_j, s, f_o)$, $Y(B_j, c, f_o)$, $Y(B_j, s, f_1)$ and $Y(B_j, c, f_1)$ have a normal distribution with means = $N\sqrt{s}\sin\varphi_o/2$, $N\sqrt{s}\cos\varphi_o/2$, $N\sqrt{s}\sin\varphi_1/2$ and $N\sqrt{s}\cos\varphi_1/2$, respectively and variance $N\sigma^2/2$. Thus from Reference 8, page 169 and Reference 4, pages 48-52 it follows that $Y^2(B_j, s, f_o)/(N\sigma^2/2)$, $Y^2(B_j, c, f_o)/(N\sigma^2/2)$, $Y^2(B_j, s, f_1)/(N\sigma^2/2)$ and $Y^2(B_j, c, f_1)/(N\sigma^2/2)$ have a non-central Chi-Squared Distribution with one degree of freedom and non-central parameters $Ns\sin^2\varphi_o/2\sigma^2$, $Ns\cos^2\varphi_o/2\sigma^2$, $Ns\sin^2\varphi_1/2\sigma^2$ and $Ns\cos^2\varphi_1/2\sigma^2$, respectively. Using the additive property we see that $E^2(B_j, f_o)/(N\sigma^2/2) = Y^2(B_j, s, f_o)/(N\sigma^2/2) + Y^2(B_j, c, f_o)/(N\sigma^2/2)$ and $E^2(B_j, f_1)/(N\sigma^2/2) = Y^2(B_j, s, f_1)/(N\sigma^2/2) + Y^2(B_j, c, f_1)/(N\sigma^2/2)$ have a non-central Chi-Squared Distribution with two degrees of freedom and non-central parameter $Ns/2\sigma^2 = N E_b/N_o$. Using the additivity property once again we see that $\mathcal{E}(B_j)/(N\sigma^2/2) = E^2(B_j, f_o)/(N\sigma^2/2) + E^2(B_j, f_1)/(N\sigma^2/2)$ has a non-central Chi-Squared Distribution with four degrees of freedom and non-central parameter $2N E_b/N_o$. Thus $\mathcal{E}(B_j)$ is determined to within the constant $C = 2/N\sigma^2$ by a non-central Chi-Squared Distribution with four degrees of freedom and non-central parameter $2N E_b/N_o$

APPENDIX 3

For the low E_b/N_o ratio case of the NCB2FSK-Add, Subtract Block Decision Technique with equi-distributed random orthogonal block encoding we investigate the distribution of $\mathcal{E}(B_j)$ assuming that the B_j block was transmitted and that the B_j block was not transmitted. In what follows we will use the symbols given in Diagram 3 with $Y(B_j, s, f_k)$ and $Y(B_j, c, f_k)$ replaced by $Z(B_j, s, f_k)$ and $Z(B_j, c, f_k)$ for $j = 0, 1$ and $k = 0, 1$ and let s be the signal power, σ^2 be the average noise power and φ_o and φ_1 be the initial, unknown signal phases at f_o and f_1, respectively.

First let us suppose that the B_j block was not transmitted. Due to the encoding employed and from Reference 1, page 168 we see that, for each of the sums $Z(B_j, s, f_o)$, $Z(B_j, c, f_o)$, $Z(B_j, s, f_1)$ and $Z(B_j, c, f_1)$, $N/4$ of the $N/2$ positive and $N/4$ of the $N/2$ negative quadrature components appearing in the sum have a normal distribution with means $= \sqrt{s} \sin \varphi_o$, $\sqrt{s} \cos \varphi_o$, $\sqrt{s} \sin \varphi_1$ and $\sqrt{s} \cos \varphi_1$ respectively and variance $= \sigma^2$ and the other $N/4$ positive and $N/4$ negative quadrature components appearing in the sum have a normal distribution with mean $= 0$ and variance $= \sigma^2$. Hence the sum of the $N/2$ quadrature components with respect to either the negative or the positive portion of each sum has from Reference 8, page 200 a normal distribution with means $= N\sqrt{s} \sin \varphi_o/4$, $N\sqrt{s} \cos \varphi_o/4$, $N\sqrt{s} \sin \varphi_1/4$ and $N\sqrt{s} \cos \varphi_1/4$ respectively and variance $N\sigma^2/2$. Hence taking into account the signs associated with each portion we see from Reference 8, pages 199-200 that $Z(B_j, s, f_o)$, $Z(B_j, c, f_o)$, $Z(B_j, s, f_1)$ and $Z(B_j, c, f_1)$ each have a normal distribution with mean $= 0$ and variance $= N\sigma^2$. Thus from Reference 8, pages 169, 180 we see that $Z^2(B_j, s, f_o)/N\sigma^2$, $Z^2(B_j, c, f_o)/N\sigma^2$, $Z^2(B_j, s, f_1)/N\sigma^2$ and $Z^2(B_j, c, f_1)/N\sigma^2$ each have a Chi-Squared Distribution with one degree of freedom. From Reference 8, page 201 it follows that $E^2(B_j, f_o)/N\sigma^2$ and $E^2(B_j, f_1)/N\sigma^2$ each have a Chi-Squared Distribution with two degrees of freedom. Using this additivity property once again it follows that $\mathcal{E}(B_j)/N\sigma^2 = E^2(B_j, f_o)/N\sigma^2 + E^2(B_j, f_1)/N\sigma^2$ has a Chi-Squared Distribution with four degrees of freedom. Thus $\mathcal{E}(B_j)$ is determined to within a constant $C = 1/N\sigma^2$ by a Chi-Squared Distribution with four degrees of freedom.

Next let us suppose that the B_j block was transmitted. In this case each of the $N/2$ positive quadrature component terms appearing in the sums $Z(B_j, s, f_o)$, $Z(B_j, c, f_o)$, $Z(B_j, s, f_1)$ and $Z(B_j, c, f_1)$ has a normal distribution with means $= \sqrt{s} \sin \varphi_o$, $\sqrt{s} \cos \varphi_o$, $\sqrt{s} \sin \varphi_1$ and $\sqrt{s} \cos \varphi_1$ respectively and variance $= \sigma^2$ and each of the $N/2$ negative quadrature component terms appearing in these sums has a normal distribution with mean $= 0$ and variance $= \sigma^2$. Thus from Reference 8, page 200 it follows that the sum of $N/2$ positive quadrature component terms in these sums have a normal distribution with means $= N\sqrt{s} \sin \varphi_o/2$, $N\sqrt{s} \cos \varphi_o/2$, $N\sqrt{s} \sin \varphi_1/2$ and $N\sqrt{s} \cos \varphi_1/2$ respectively and variance $= N\sigma^2/2$ and the sum of the $N/2$ negative quadrature component terms in these sums have a normal distribution with mean $= 0$ and variance $= N\sigma^2/2$. Hence taking into account the signs associated with each type of sum we see from Reference 8, pages 199 and 200 that $Z(B_j, s, f_o)$,

$Z(B_j, c, f_o)$, $Z(B_j, s, f_1)$ and $Z(B_j, c, f_1)$ have a normal distribution with means = $N\sqrt{s} \sin \varphi_o/2$, $N\sqrt{s} \cos \varphi_o/2$, $N\sqrt{s} \sin \varphi_1/2$ and $N\sqrt{s} \cos \varphi_1/2$ respectively and variance = $N\sigma^2$. Thus from Reference 3, page 169 and Reference 4, pages 48-52 it follows that $Z(B_j, s, f_o)/N\sigma^2$, $Z(B_j, c, f_o)/N\sigma^2$, $Z(B_j, s, f_1)/N\sigma^2$ and $Z(B_j, c, f_1)/N\sigma^2$ have a non-central Chi-Squared Distribution with one degree of freedom and non-central parameters $Ns \sin^2 \varphi_o/4$, $Ns \cos^2 \varphi_o/4$, $Ns \sin^2 \varphi_1/4$ and $Ns \cos^2 \varphi_1/4$ respectively. Using the additivity property we see that $E^2(B_j, f_o)/N\sigma^2 = Z^2(B_j, s, f_o)/N\sigma^2 + Z^2(B_j, c, f_o)/N\sigma^2$ and $E^2(B_j, f_1)/N\sigma^2 = Z^2(B_j, s, f_1)/N\sigma^2 + Z^2(B_j, c, f_1)/N\sigma^2$ have a non-central Chi-Squared Distribution with two degrees of freedom and non-central parameter $Ns/4 = N E_b/2N_o$. Using the additivity property once again we see that $\mathcal{E}(B_j)/N\sigma^2 = E^2(B_j, f_o)/N\sigma^2 + E^2(B_j, f_1)/N\sigma^2$ has a non-central Chi-Squared Distribution with four degrees of freedom and non-central parameter $N E_b/N_o$. Thus $\mathcal{E}(B_j)$ is determined to within the constant $C = 1/N\sigma^2$ by a non-central Chi-Squared Distribution with four degrees of freedom and non-central parameter $N E_b/N_o$.

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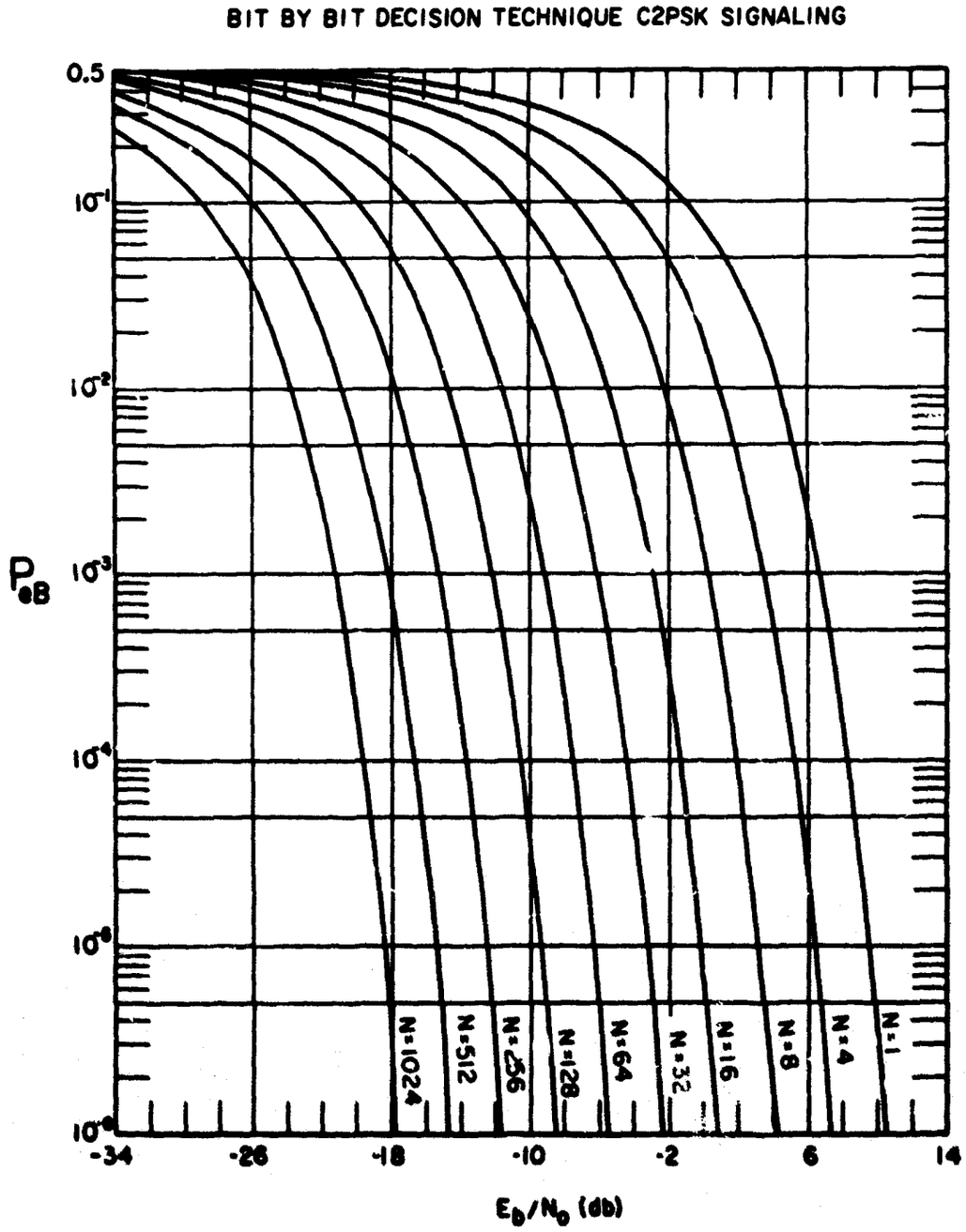
BIT BY BIT DECISION TECHNIQUE C2PSK SIGNALING
0.5
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
P_{eB}
N=1024
N=512
N=256
N=128
N=64
N=32
N=16
N=8
N=4
N=1
-34
-26
-18
-10
-2
6
14
E_b/N_o (db)

Figure 1

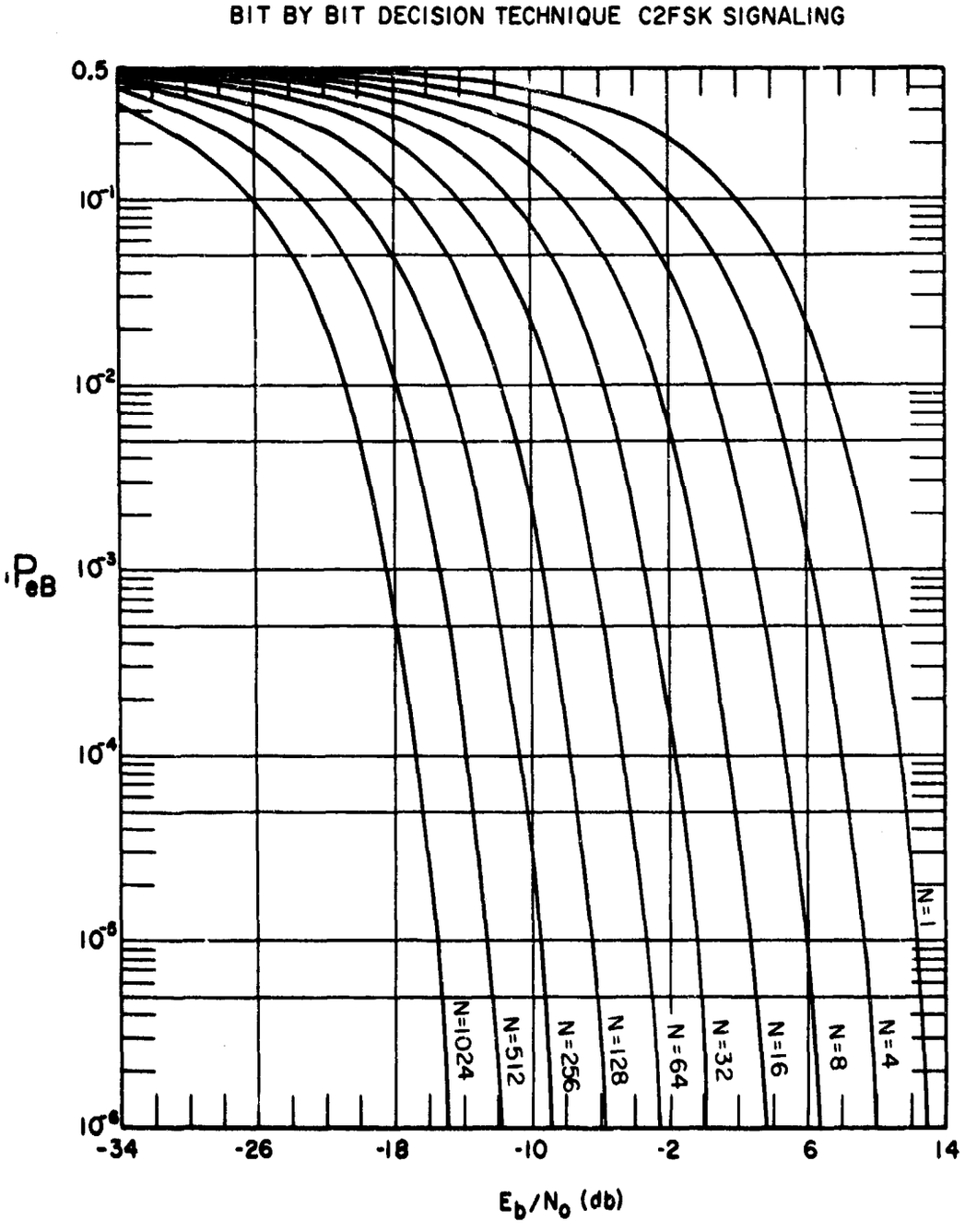
BIT BY BIT DECISION TECHNIQUE C2FSK SIGNALING
0.5
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
P_{eB}
-34
-26
-18
-10
-2
6
14
E_b/N_0 (db)
N=1
N=4
N=8
N=16
N=32
N=64
N=128
N=256
N=512
N=1024

Figure 2

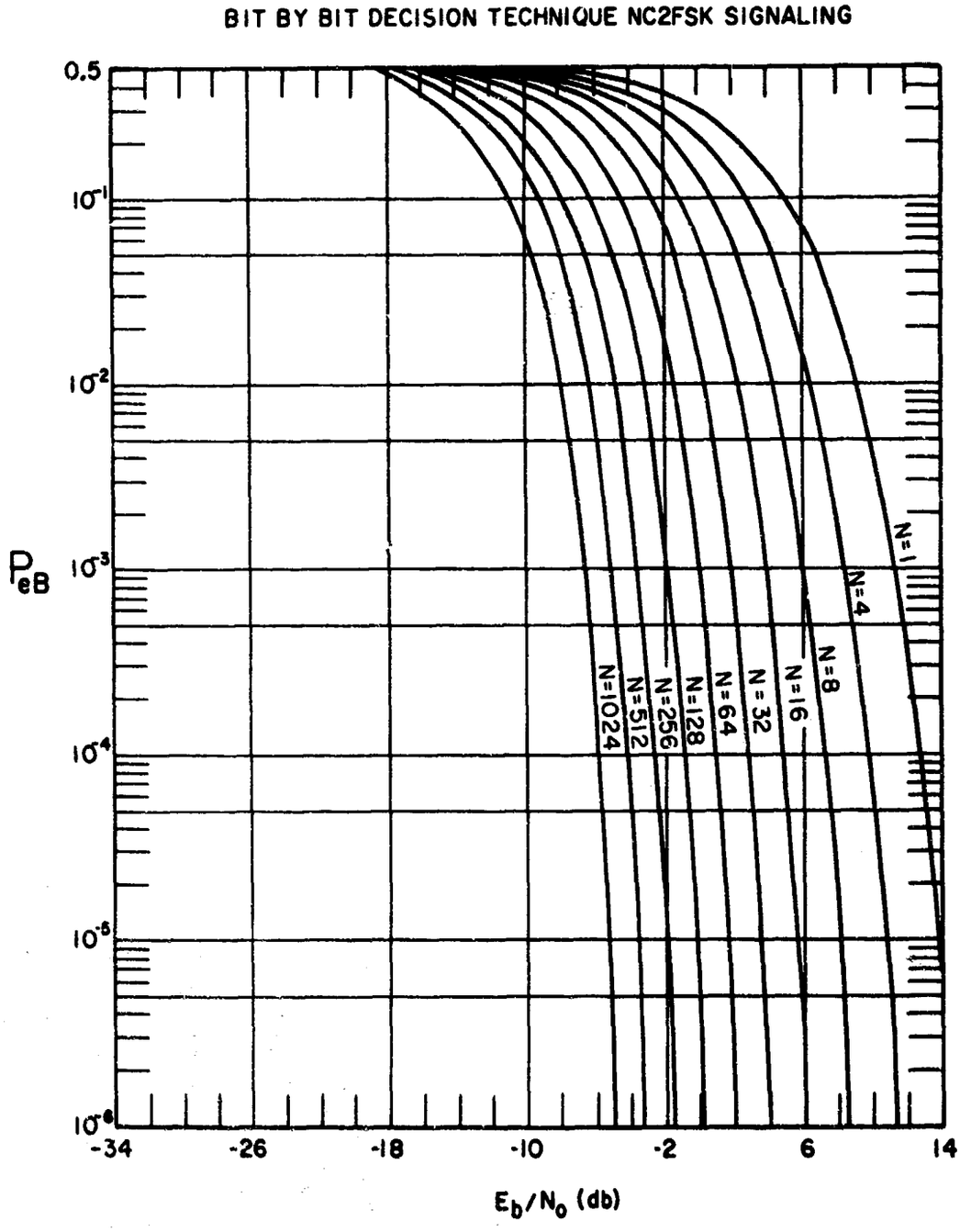
BIT BY BIT DECISION TECHNIQUE NC2FSK SIGNALING
0.5
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
P_{eB}
N=1
N=4
N=8
N=16
N=32
N=64
N=128
N=256
N=512
N=1024
-34
-26
-18
-10
-2
6
14
E_b/N_o (db)

Figure 3

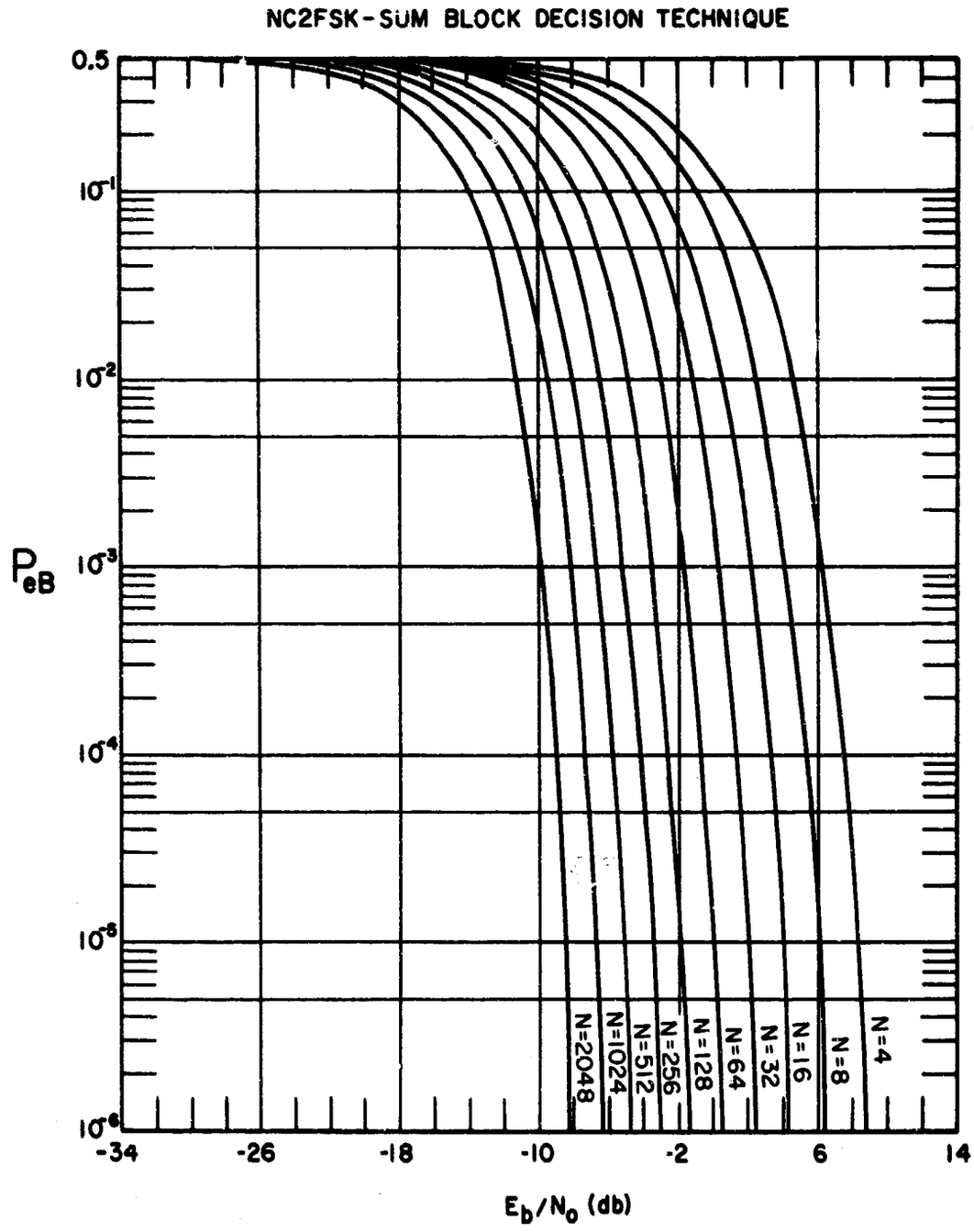
NC2FSK-SUM BLOCK DECISION TECHNIQUE
0.5
10⁻¹
10⁻²
10⁻³
10⁻⁴
10⁻⁵
10⁻⁶
P_eB
-34
-26
-18
-10
-2
6
14
E_b/N_0 (db)
N=4
N=8
N=16
N=32
N=64
N=128
N=256
N=512
N=1024
N=2048

Figure 4

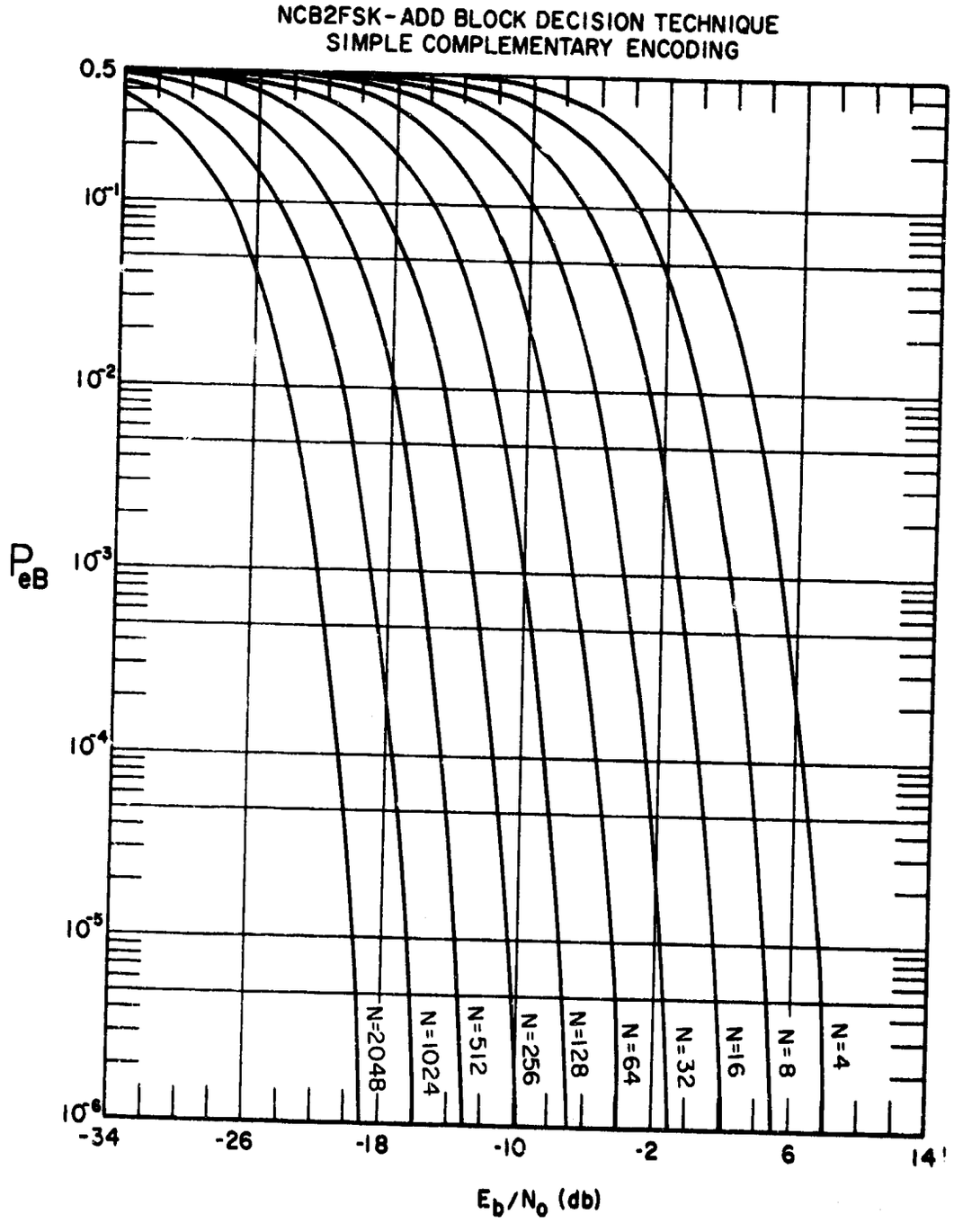
NCB2FSK-ADD BLOCK DECISION TECHNIQUE
SIMPLE COMPLEMENTARY ENCODING
0.5
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
P_{eB}
N=2048
N=1024
N=512
N=256
N=128
N=64
N=32
N=16
N=8
N=4
-34
-26
-18
-10
-2
6
14
E_b/N_0 (db)

Figure 5

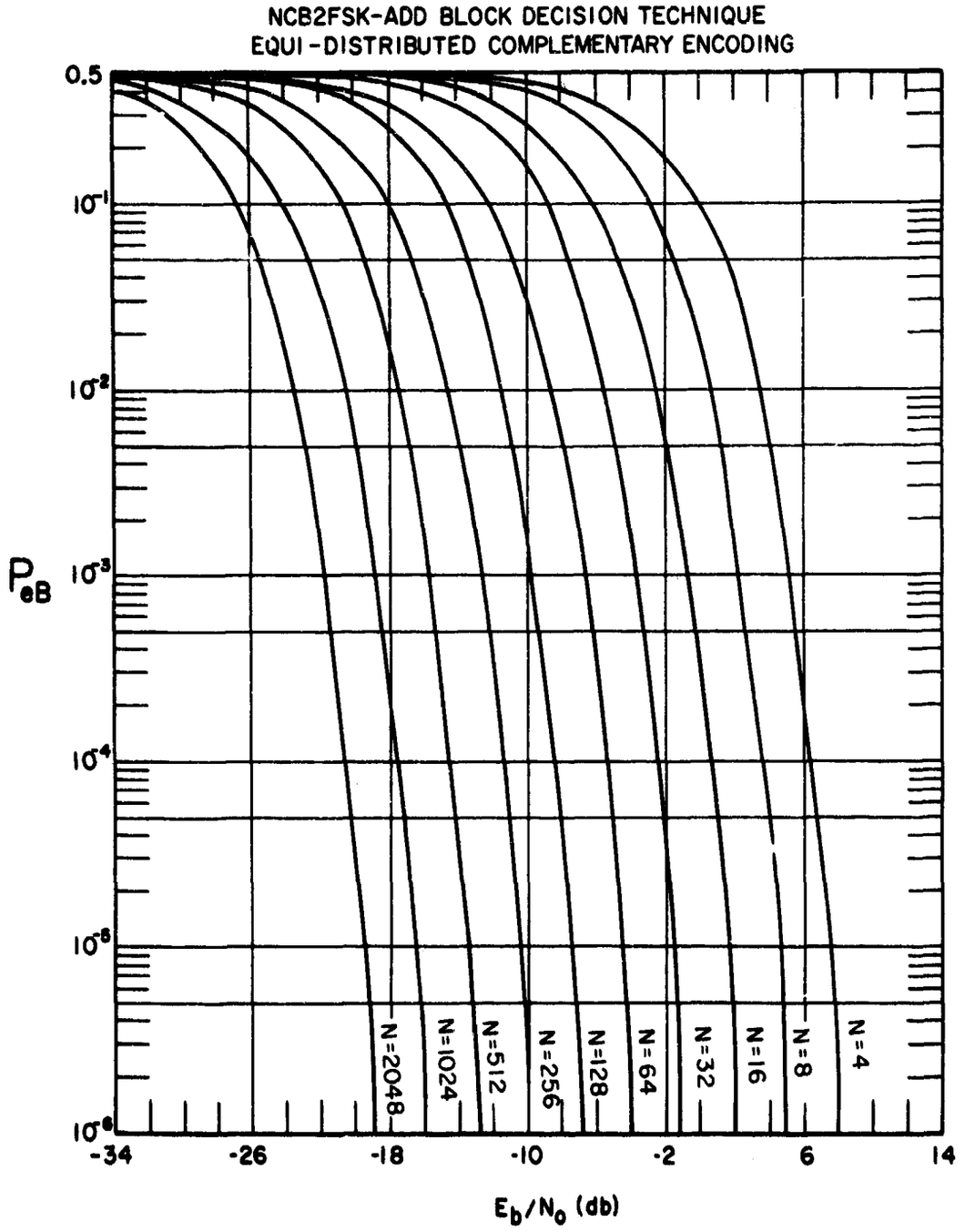
NCB2FSK-ADD BLOCK DECISION TECHNIQUE
EQUI-DISTRIBUTED COMPLEMENTARY ENCODING
0.5
10⁻¹
10⁻²
10⁻³
10⁻⁴
10⁻⁵
10⁻⁶
P_eB
N=2048
N=1024
N=512
N=256
N=128
N=64
N=32
N=16
N=8
N=4
-34
-26
-18
-10
-2
6
14
E_b/N_0 (db)

Figure 6

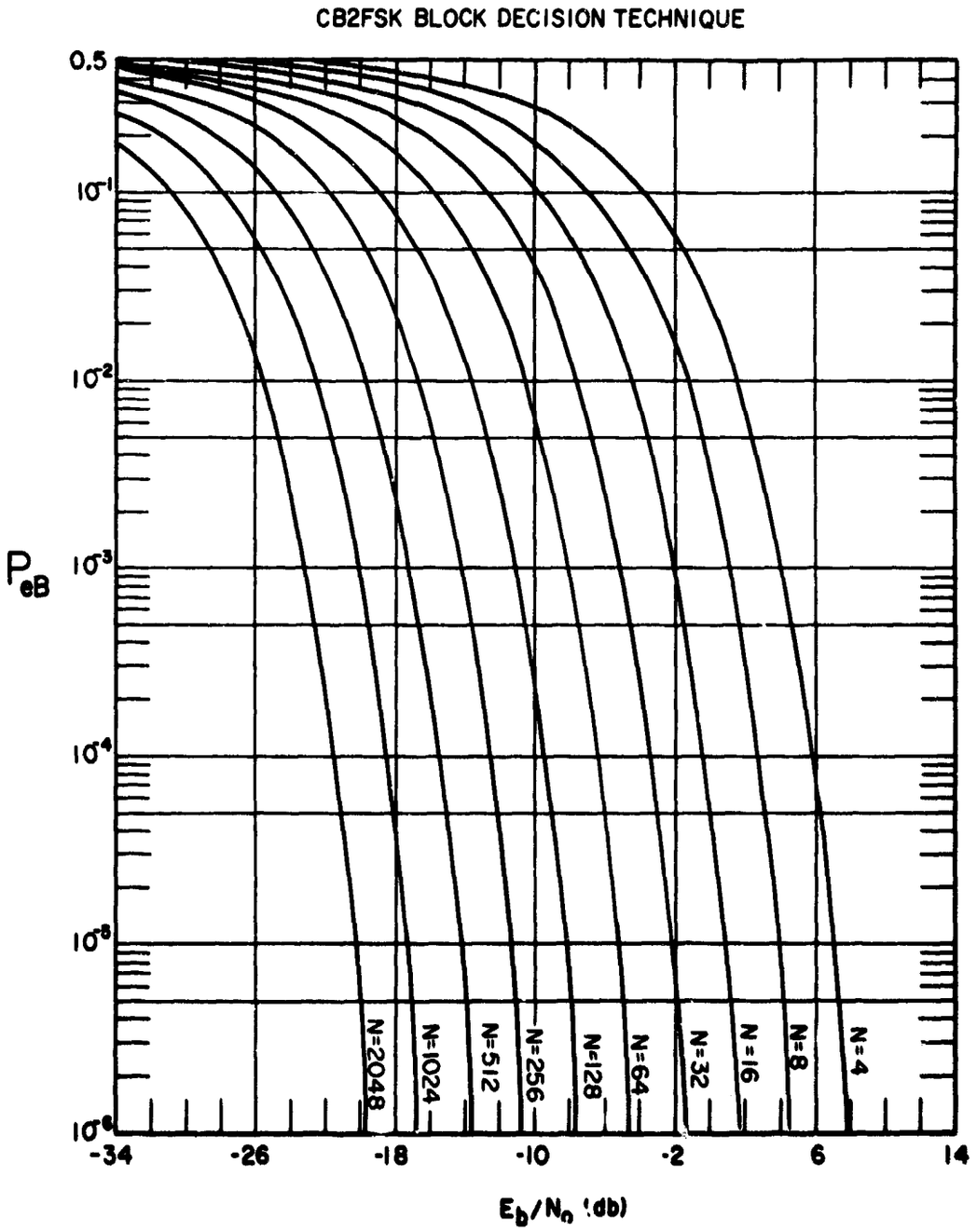
CB2FSK BLOCK DECISION TECHNIQUE
0.5
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
P_{eB}
N=2048
N=1024
N=512
N=256
N=128
N=64
N=32
N=16
N=8
N=4
-34
-26
-18
-10
-2
6
14
E_b/N_o (db)

Figure 7

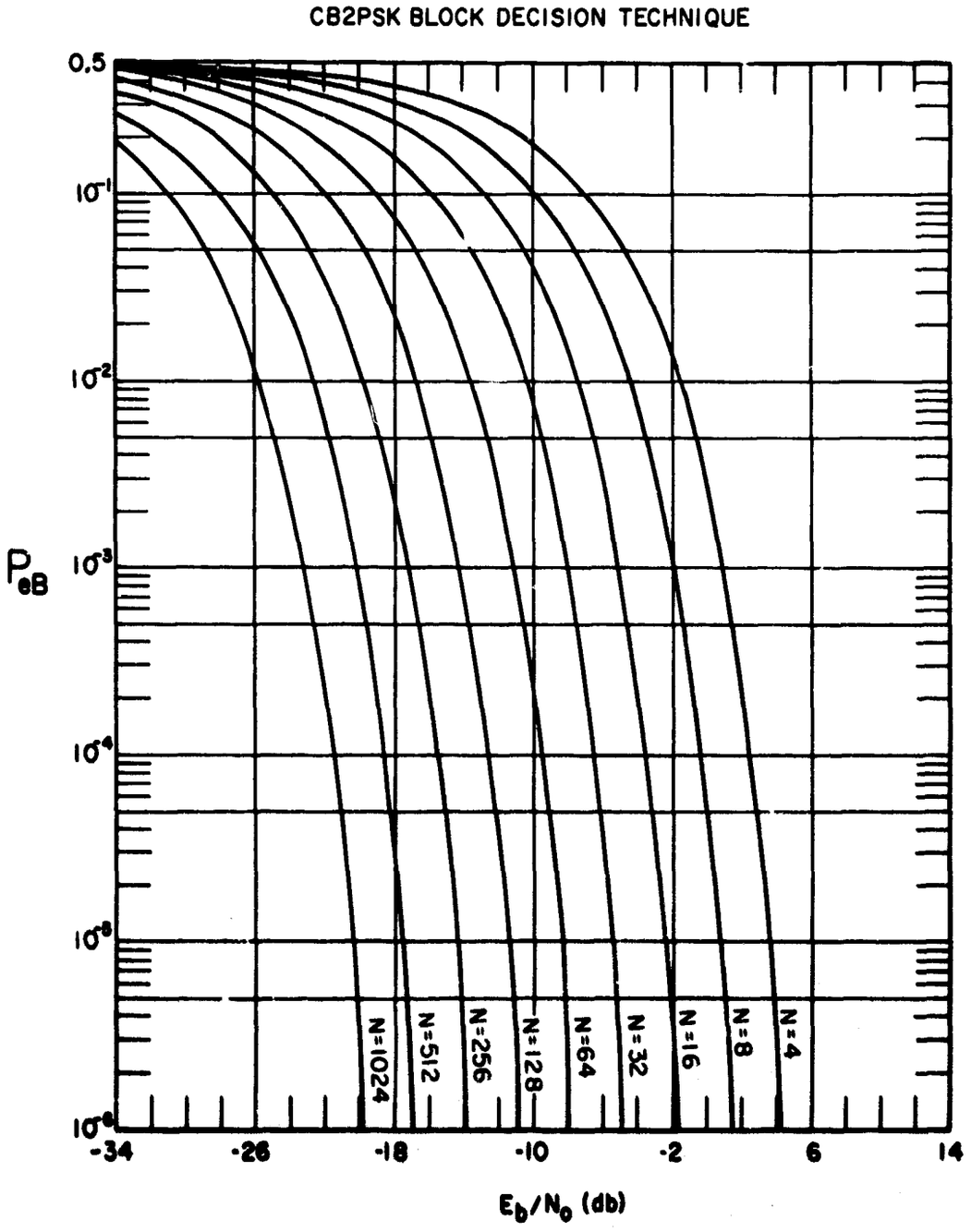
CB2PSK BLOCK DECISION TECHNIQUE
0.5
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
P_{eB}
N=1024
N=512
N=256
N=128
N=64
N=32
N=16
N=8
N=4
-34
-26
-18
-10
-2
6
14
E_b/N_0 (db)

Figure 8

BIT BY BIT DECISION TECHNIQUE RECEIVER
(NC2FSK SIGNALING)

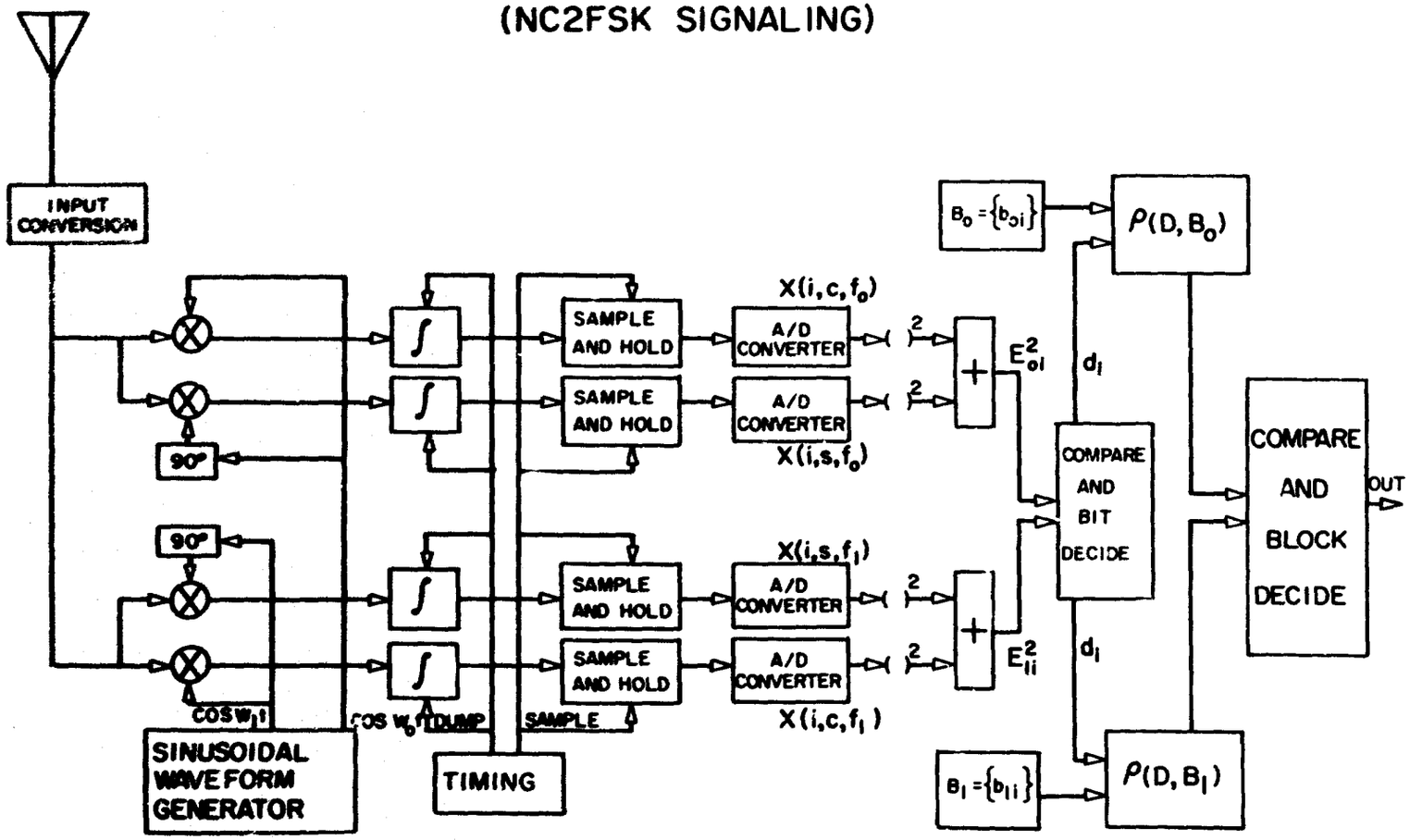

Diagram 1

NC2FSK-SUM BLOCK DECISION TECHNIQUE RECEIVER

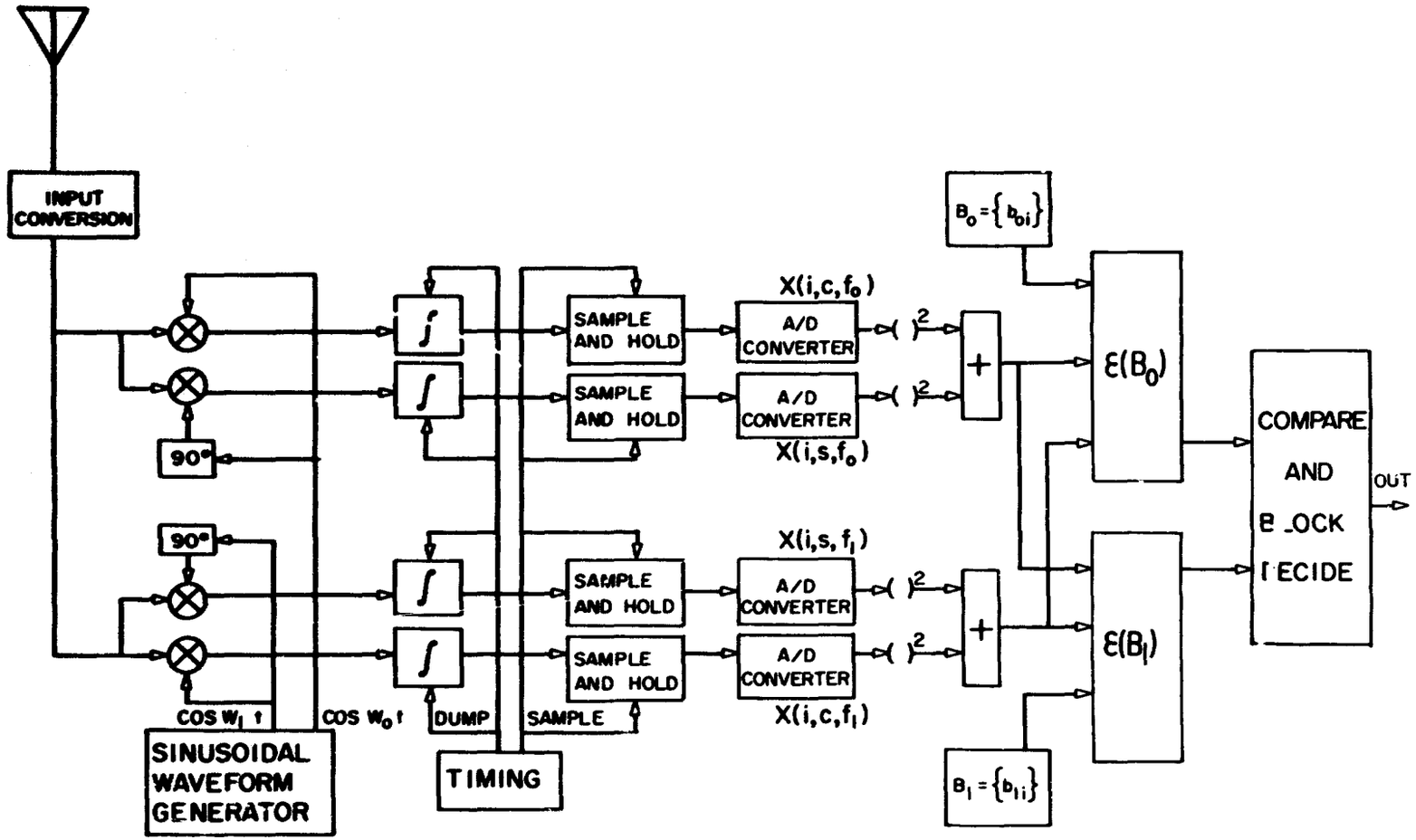

Diagram 2

NCB2FSK-ADD BLOCK DECISION TECHNIQUE RECEIVER

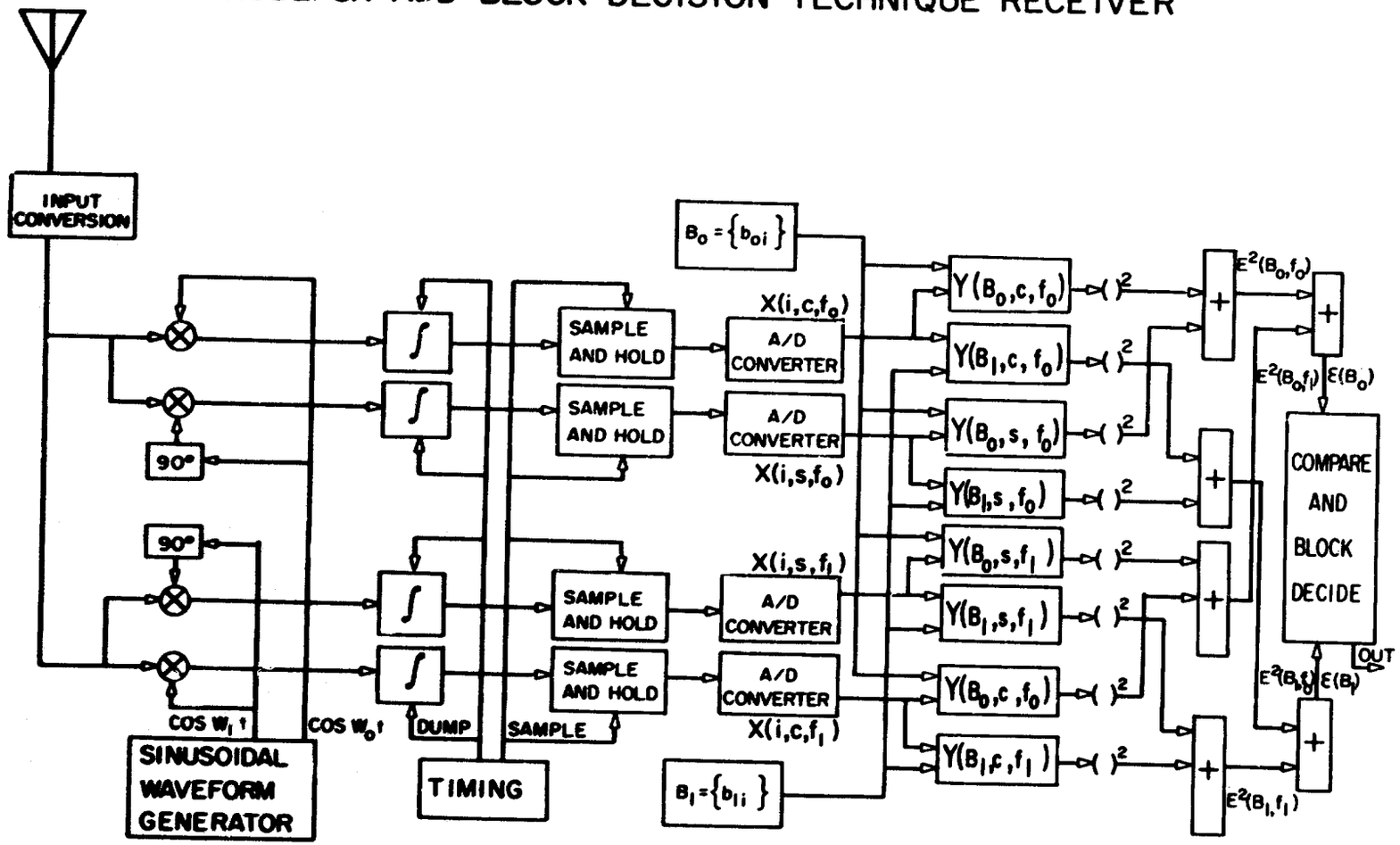

Diagram 3

INTELLIGENT PORTION OF A RECEIVER
(NC2FSK SIGNALING)

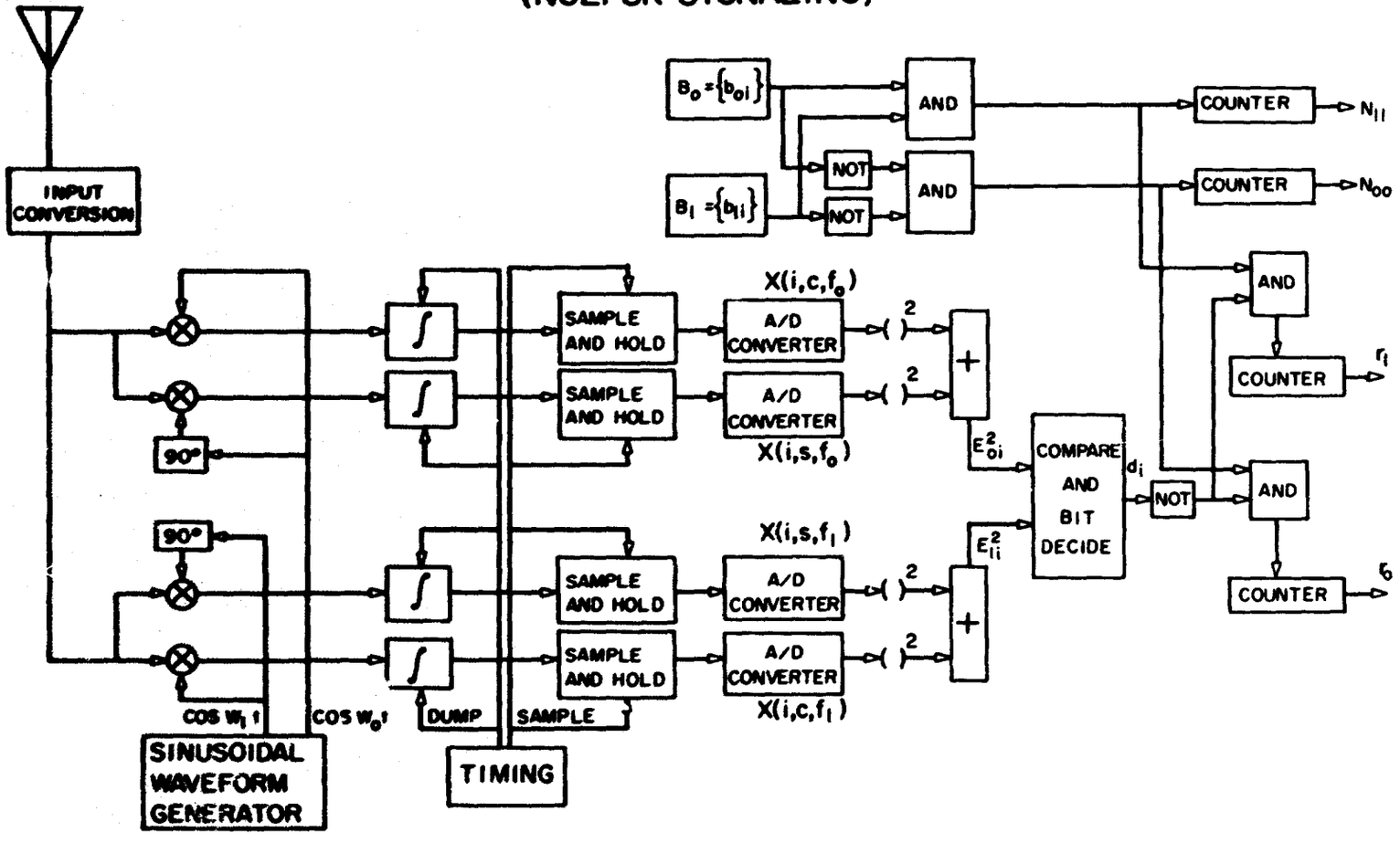

Diagram 4

Security Classification

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13. ABSTRACT

Various types of fixed length binary block encodings of binary data are presented. Binary signaling is used to transmit the binary encoded data through an additive white gaussian noise channel. Certain block decision techniques are investigated and compared. For certain block encoding schemes, the notion of an intelligent receiver is developed.

This study is considered basic in analyzing the use of redundant encoding in digital communications systems.

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Security Classification

14 KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Redundancy						
Binary block code						
Block encoding scheme						
Correlation						
Coherent binary signaling						
Incoherent binary signaling						
Gaussian channel						
Block decision technique						
Bit by bit detection						
Post-detection integration						
Predetection integration						
Probability of a block error (PeB)						
Intelligent receiver						

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